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Instability of two-dimensional Ising ferromagnets with dipole interactions

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Abstract. Two-dimensional Ising ferromagnets with spins perpendicular to the plane are destabilised by dipole interactions. Ferromagnetism is destroyed by domain walls which, at low temperatures, form a square network on a square lattice. At higher temperatures, domain walls form a floating solid, the period of which decreases continuously with increasing temperature.

1. Introduction

It was recently shown by Yafet and Gyorgy (YG) (1988) that ferromagnetic ultra-thin films with an easy axis perpendicular to the film are unstable with respect to domain formation if the anisotropy is larger than some threshold. The case treated by YG was that of a weak anisotropy and a weak dipole interaction, as relevant for Ni, Fe or Gd films.

Here, we present a study of the opposite limit: infinite anisotropy *perpendicular* to the film and dipole interaction g not very much weaker than the exchange coupling J, say $10 < J/g < 20$. The results are expected to hold qualitatively even for reasonable anisotropy. If J/g becomes too large, the ground-state domain size increases exponentially and becomes rapidly larger than any laboratory—at least in the Ising limit.

The following results will be established.

(i) The domain size, as already noted by YG, is independent of the size and shape of the sample. Thus, in contrast with the three-dimensional case, domains cannot be suppressed by using a toroidal sample. Thus, it is more appropriate to speak of a magnetic structure than of domains.

(ii) The domain size is finite (although usually large) even for an infinite anisotropy in contrast with the results of YG.

(iii) The domain size is a function of temperature. Near the transition, it is moderate even when J/g is large.

(iv) The equilibrium magnetic structure is made of intersecting domain walls in several directions.

(v) Except at very low temperatures or for very low values J/g (<2) the walls form

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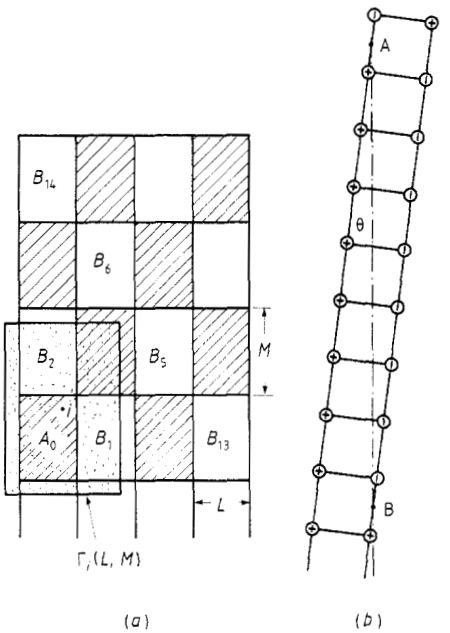

Figure 1. (*a*) A rectangular domain structure. If $i \in A_0$, the sites j contained in the shaded rectangles do not contribute to (4). The dotted rectangle is $\Gamma_i(L, M)$. (*b*) The domain wall segment AB has length $1/\sin\theta$ and cuts $1/\tan\theta$ vertical bonds and one horizontal bond. Its energy per length is therefore $2J(1 + \cot\theta)\sin\theta$ in agreement with (3).

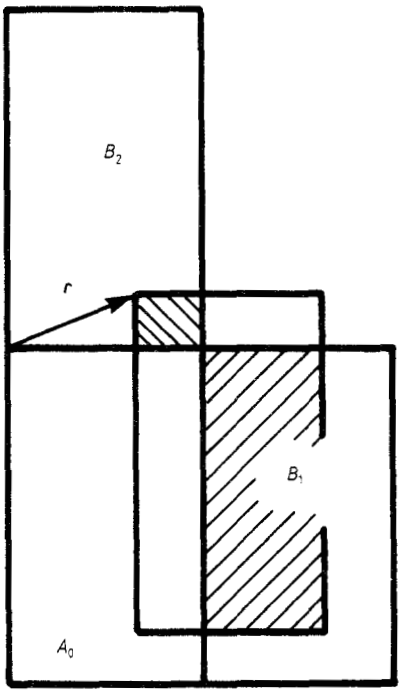

Figure 2. Geometrical significance of $n_m(\boldsymbol{r})$ in (8).

a 'floating solid' of the type discussed by Jancovici (1967), Kosterlitz and Thouless (1973) and Nelson and Halperin (1979).

These properties seem to be new in spite of previous work on domain walls in thin films (Néel 1955) and on magnetic structures of dipolar magnets (Capel 1965, Kretschmer and Binder 1979).

2. Magnetic structure at $T = 0$

We assume that the magnetic structure at $T = 0$ consists of a regular array of domains with respective + and − spins, e.g. a rectangular array (figure 1). The rectangular structure with sides L and M includes as special cases the square network ($L = M$) and the striped network ($M = \infty$). It is difficult to exclude completely the possibility of more complicated structures, but it will be shown that the ground state is neither ferromagnetic nor striped.

For definiteness the case of a square lattice with lattice constant of unity and of a rectangular wall structure will be considered.

If the ferromagnetic structure is taken to have zero energy, the energy of a rectangular wall network with sides L and M is

$$W = W_{ex} + W_{dip} \tag{1}$$

where the exchange part for a crystal of N atoms is

$$W_{ex} = 2J(N/LM)(L + M)f(\theta). \tag{2}$$

θ is the angle of the walls with the directions of the square unit cell. An elementary calculation yields (figure 1(*b*))

$$f(\theta) = |\cos\theta| + |\sin\theta|. \tag{3}$$

The dipole part is

$$W_{dip} = -\tfrac{1}{2}\sum_{ij=1}^{N} \frac{g}{r_{ij}^3}(1 - \sigma_i\sigma_j) \tag{4}$$

where i, j denote atoms with spins σ_i, $\sigma_j = \pm 1$. One can restrict i to a given cell of the wall network, denoted A_0, and multiply by the number $\nu = N/LM$ of cells. Now $1 - \sigma_i\sigma_j = 0$ for one half of the cells (including A_0) and $1 - \sigma_i\sigma_j = 2$ for the other cells $B_1, B_2, \ldots$:

$$W_{dip} = -\frac{N}{LM} g \sum_{i\in A_0} \sum_{m}^{\nu/2} \sum_{j\in B_m} \frac{1}{r_{ij}^3} = -\frac{N}{LM} g \sum_{m=1}^{\nu/2} K_m \tag{5}$$

where

$$K_m = \sum_{i\in A_0} \sum_{j\in B_m} \frac{1}{r_{ij}^3}. \tag{6}$$

It is shown in the Appendix that, if g/J is small enough, the contribution W''_{dip} to (5) of those cells B_m which are *not* neighbours to A_0 is negligible with respect to (2), and therefore cannot destabilise the ferromagnetic structure. This property is easily extended to other types of crystal lattice and wall network. We now consider the contributions to (5) of the four neighbouring cells B_1, B_2, B_3 and B_4 (figure 1):

$$W'_{dip} = -2(N/LM)g(K_1 + K_2). \tag{7}$$

For given m, $\boldsymbol{r}_{ij}$ has certain allowed values $\boldsymbol{r}$. Let $n_m(\boldsymbol{r})$ be the number of pairs $i(\in A_0)$, $j(\in B_m)$ with a given $\boldsymbol{r}$. Then

$$K_m = \sum_{\boldsymbol{r}} \frac{n_m(\boldsymbol{r})}{r^3}. \tag{8}$$

We now assume that

$$L, M \gg 1. \tag{9}$$

Then, $n_m(\boldsymbol{r})$ is the intersection of B_m with $T(\boldsymbol{r})A_0$ (figure 2), where $T(\boldsymbol{r})$ is the projection operator with translation vector $\boldsymbol{r} = (x, y)$. For the two adjacent cells K_1 and K_2 (figure 2) it is readily seen that n_1 and n_2 are zero except in the following cases:

$$\begin{aligned}
&n_1(\boldsymbol{r}) = (M - |y|)x && 0 \leqslant x \leqslant L, && -M \leqslant y \leqslant M\\
&n_1(\boldsymbol{r}) = (M - |y|)(2L - x) && L \leqslant x \leqslant 2L, && -M \leqslant y \leqslant M\\
&n_2(\boldsymbol{r}) = (L - |x|)y && 0 < y < M, && -L \leqslant x \leqslant L\\
&n_2(\boldsymbol{r}) = (L - |x|)(2M - y) && M < y < 2M, && -L \leqslant x \leqslant L.
\end{aligned}$$

In the limit (9), the sum (8) can be replaced by an integral:

$$K_1 = 2\int_{y_0}^{M} \mathrm{d}y \int_{x_0}^{L} x\,\mathrm{d}x \frac{M - y}{(x^2 + y^2)^{3/2}} + 2\int_{y_0}^{M} \mathrm{d}y \int_{L}^{2L} \mathrm{d}x\,(2L - x)\frac{M - y}{(x^2 + y^2)^{3/2}} \tag{10}$$

with appropriate lower integration bounds x_0 and y_0. As it is shown in the Appendix, at

least the leading behaviour of K_1 for $L, M \gg 1$ does not depend on these bounds. The result of the calculation (see Appendix) is

$$K_1 = 2M \ln[L/(1 + \sqrt{1 + L^2/M^2})] + O(\sqrt{L^2 + M^2}). \tag{11}$$

If g/J is small, only the first term can balance the exchange energy (2). K_2 is obtained from (11) by interchanging L and M. Insertion of (2), (7) and (11) into (1) yields, as seen:

$$W/N = 2J(L^{-1} + M^{-1})f(\theta) - (4/L)g \ln L - (4/M)\, g \ln M + (g/LM)O(\sqrt{L^2 + M^2}). \tag{12}$$

Minimisation with respect to θ yields at once the result that $f(\theta)$ should be minimum. Hence, $\theta = 0$, domain walls should be parallel to the sides of the cubic unit cell and $f(\theta) = f(0) = 1$. Replacing $f(\theta)$ by 1, minimisation of (12) with respect to L and M yields for $L, M \gg 1$ (i.e. for J sufficiently larger than g)

$$L = M = \exp(1 + J/2g). \tag{13}$$

This increases from 400 to 60 000 inter-atomic distances when J/g goes from 10 to 20. Since the last term in (12) is neglected—which is justified only for $J/g \gg 1$—equation (13) and thus the above values should only be considered as estimates of the order of magnitude of L. If J/g takes the high value that it has in Gd or Fe, L and M may be in practice replaced by ∞, as found by YG using continuous spins instead of an Ising model.

The conclusion of this section is that, of all rectangular structures, the square network of domain walls parallel to the sides of the square unit cell has the lowest energy. Presumably, it is the ground state. All structures with domain walls parallel to other directions are expected to have a higher energy because of the factor $f(\theta)$ in (2).

3. Effect of temperature

The evaluation (13) holds so long as the domain wall thickness is of the order of magnitude of the inter-atomic distance. In the Ising model, this property is satisfied below some fraction of T_c, at least below $T_c/2$ in any case.

It will now be seen that, in the mean-field approximation, the period becomes much smaller (for $J \gg g$) near T_c.

In the mean-field approximation (Villain 1962) the thermal averages $m_i = \langle \sigma_i \rangle$ should minimise the free-energy functional

$$F = -\sum_{ij} \tilde{J}_{ij} m_i m_j + \tfrac{1}{2} K_B T \sum_i [(1 + m_i) \ln(1 + m_i) + (1 - m_i) \ln(1 - m_i)] \tag{14}$$

where

$$\tilde{J}_{ij} = J - g/r_{ij}^3 \qquad \text{if } i, j \text{ are neighbours} \tag{15a}$$

$$\tilde{J}_{ij} = -g/r_{ij}^3 \qquad \text{otherwise.} \tag{15b}$$

Introducing the Fourier transforms $\tilde{m}_k$ and expanding the logarithms, (14) reads

$$F = \sum_k [K_B T - \tilde{J}(\boldsymbol{k})] \tilde{m}_k \tilde{m}_{-k} + \frac{K_B T}{6} \sum_i m_i^4 + \ldots \tag{16}$$

where

$$\tilde{J}(\boldsymbol{k}) = \sum_j \tilde{J}_{ij} \cos(\boldsymbol{k} \cdot \boldsymbol{r}_{ij}) = 2J(\cos k_x + \cos k_y) - g \sum_j r_{ij}^{-3} \cos(\boldsymbol{k} \cdot \boldsymbol{r}_{ij}). \tag{17}$$

The minimum of (16) corresponds to $\tilde{m}_k \equiv 0$ for $K_B T > \tilde{J}(\boldsymbol{k}_M)$, where $\boldsymbol{k}_M$ is the value of $\boldsymbol{k}$ for which $J(\boldsymbol{k})$ takes its maximum.

At the temperature

$$T_c = \tilde{J}(\boldsymbol{k}_M)/K_B \tag{18}$$

a transition occurs, which is continuous since (16) contains no third-order terms and the fourth-order terms have positive coefficients. The period $2L$ just below T_c is given by

$$L(T_c) = \pi/k_M. \tag{19}$$

There is in principle no difficulty in determining (for instance numerically) the minimum of (14) at all temperatures. This will not be done, and only the neighbourhood of T_c will be explicitly treated. The remaining task is to evaluate k_M.

We asume that $L \gg 1$ (i.e. $k \ll 1$). In this case the sum on the right-hand side of (17),

$$S(\boldsymbol{k}) = \sum_{r \neq 0} \frac{1}{r^3} \cos(\boldsymbol{k} \cdot \boldsymbol{r})$$

is nearly isotropic with respect to $\boldsymbol{k}$. For convenience, we choose $\boldsymbol{k}$ to be parallel to a side of the cubic unit cell (x direction). Then, $S(\boldsymbol{k})$ is given by

$$S(\boldsymbol{k}) = 2 \sum_{x=1}^{\infty} \cos(kx) R(x) + 2 \sum_{y=1}^{\infty} \frac{1}{y^3} \tag{20}$$

where $R(x)$ is the sum over an atomic row at distance x perpendicular to the direction of $\boldsymbol{k}$. This sum can be replaced by an integral if x is not too small:

$$R(x) = \sum_{y=-\infty}^{\infty} \frac{1}{(x^2 + y^2)^{3/2}} \simeq \int_{-\infty}^{\infty} \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{2}{x^2}.$$

This approximation that has been discussed in detail by YG. The corresponding error is only of the order of 1% for the smallest possible $x = 1$. Now the sum over x may also be replaced by an integral if one introduces an appropriate lower integration bound x_0:

$$\sum_{x=1}^{\infty} \frac{1}{x^2} \cos(kx) \to \int_{x_0}^{\infty} \frac{dx}{x^2} \cos(kx) = \frac{\cos(kx_0)}{x_0} + k\,\mathrm{Si}(kx_0) - k\frac{\pi}{2}. \tag{21}$$

Since we are interested in small values of k, it is a reasonable approximation to choose the bound $x_0 = 6/\pi^2$ which yields the correct result for $k = 0$ (cf. the work by YG). With $c = \Sigma_{y=1}^{\infty} y^{-3} \simeq 1.202$, insertion of (21) into (20) finally yields

$$\tilde{J}(\boldsymbol{k}) = 2J(\cos k_x + \cos k_y) - 4g\{[\cos(kx_0)]/x_0 + k\,\mathrm{Si}(kx_0)\} + 2\pi g k - 2c$$
$$\underset{k \ll 1}{\sim} 4J - 2g(2/x_0 + c) + 2\pi g k - (J + 2gx_0)k^2 + O(k^3).$$

The maximum corresponds to $k_M = \pi g/(J + 2gx_0)$, i.e. with (19)

$$L(T_c) = (J + 2gx_0)/g \simeq J/g \tag{22}$$

which is considerably smaller than (13), and physically meaningful even for large values of J/g. Unfortunately, (22) is only a mean-field approximation!

When the temperature is decreased from T_c to 0, L increases from (22) to (13). This occurs continuously except at very low temperatures as seen in § 4 which corrects the mean-field approximation. This correction is necessary because the system may be regarded as a two-dimensional solid of domain walls. A two-dimensional solid is known to have infinitely large position fluctuations (Jancovici 1967) so that the mean-field approximation is not reliable. Note that the transition temperature has no reason to be affected by wall fluctuations. The transition is determined by the appearance of a finite domain magnetisation. Thus, in the limit $g \ll J$, T_c has approximately the same value as

in the *ferromagnetic Ising* model with interactions J between nearest neighbours, which has the same order of magnitude as (18).

4. Floating and commensurate phases

In (12), there is no interaction term between L and M (i.e. between walls of both directions at leading order). We shall assume that this interaction can also be neglected in excited states and we shall consider only one family of walls, say those parallel to y, and ignore the interaction with walls parallel to x.

If dipole interactions are provisionally forgotten, walls should be allowed to have kinks $\delta x = \pm 1$, the average distance of which is of the order of (Villain *et al* 1985, Villain and Bak 1981)

$$l = \exp(2\beta J). \tag{23}$$

Since walls are straight on distances much shorter than l, a wall (say the mth wall) is still reasonably well determined if one does not know all its points $x_m(y)$, but only the intersections $x_m^p = x_m(pl')$. l' should be chosen sufficiently smaller than l (say $l' = l/3$) so that $|x_m^p - x_m^{p-1}| \leqslant 1$ with a probability close to unity.

Forgetting interaction between walls, the free energy can easily be evaluated as a function of the x_m^p values since it is a one-dimensional problem. Let us consider, for example, the piece of the mth wall between $y = (p-1)l'$ and $y = pl'$ and define $\Delta_m^p = x_m^p - x_m^{p-1}$. Because of our choice of l' it should be sufficient to consider only configurations of the piece of wall with at most one kink $\delta x = \pm 1$ somewhere in between. The possible values for Δ_m^p are then given by 0 and ± 1. For each value of Δ_m^p, one may average over all internal wall configurations which correspond to that particular Δ_m^p and thus introduce the free energy $F'(\Delta_m^p)$ of the piece of wall. Under the above assumption, F' is easily determined:

$$\exp[-\beta F'(\Delta)] = \begin{cases} 1 & \Delta = 0 \\ l' \exp(-2\beta J) = l'/l & \Delta = \pm 1. \end{cases}$$

Note that $2J$ is the energy of a kink. The factor l' corresponds to the l' possible locations of a kink on the piece of wall under consideration. Note that the error introduced by neglecting configurations with two or more kinks is of order $(l/l')^2$. F' can be written as

$$F'(\Delta) = K_B T \Delta^2 \ln(l/l').$$

Thus the total free energy is

$$F_{ex}(\{x_m^p\}) = \sum_{mp} F'(\Delta_m^p) = K_B T \ln\left(\frac{l}{l'}\right) \sum_{mp} (x_m^p - x_m^{p-1})^2. \tag{24}$$

Now, we want to take dipole interactions into account. We would like to write the interaction as a sum of effective interactions $V_m^p(x_{m'}^{p'} - x_{m'+m}^{p'+p})$ between pieces of walls. We consider low temperatures where l is large, so that one can keep the terms $p = 0$ only. However, it is proved in the Appendix that only neighbouring domains contribute to the dominant terms in (12). So we keep only the terms $m = 1$ and write the dipole energy as

$$\mathcal{H}_{dip} = \sum_{mp} V(x_m^p - x_{m-1}^p).$$

V is the product of the number l' of atoms per degree of freedom, by the domain wall

dipole energy per atom, which is $-4g \ln L$ according to (12). Writing $x_m^p = mL + u_m^p$, one finds that

$$V(x_m^p - x_{m-1}^p) \simeq l'[V(L) + 2gL^{-2}(u_m^p - u_{m-1}^p)^2].$$

Thus, together with (24)

$$F(\{x_m^p\}) = \text{constant} + K_B T \ln\left(\frac{l}{l'}\right) \sum_{mp} (u_m^p - u_m^{p-1})^2 + 2gl'L^{-2} \sum_{mp} (u_m^p - u_{m-1}^p)^2. \tag{25}$$

This is an anisotropic SOS Hamiltonian (Burton and Cabrera 1949, Chui and Weeks 1976, van Beijeren and Nolden 1987). This Hamiltonian is known to have a transition ('roughening transition') at a temperature T_R above which $\langle (u_m^p)^2 \rangle$ is infinite in an infinite sample while it is finite below T_R. In this present context, a finite value of $\langle (u_m^p)^2 \rangle$ means that the domain wall network is pinned by the lattice. An estimation for T_R is easily performed if one remarks that, for $l/l' \simeq 3$, one coupling constant is of order $K_B T$. Now, if one chooses T such that the other coupling constant is of order $K_B T$ as well, this temperature is close to T_R since the model is then roughly isotropic, and in the isotropic SOS model $K_B T_R$ is approximately equal to the coupling constant. Thus, $K_B T_R \simeq 2gL^{-2}l'(T_R) \simeq gL^{-2}l(T_R)$, or according to (23)

$$K_B T_R \simeq gL^{-2} \exp(J/K_B T_R). \tag{26}$$

We are mainly interested in the case $g \ll J$. Let us make the *ansatz* $T_R \ll T_c$. Then L is given by (13) and insertion into the above equation yields

$$K_B T_R \simeq g. \tag{27}$$

Comparison with (18) shows that the *ansatz* $T_R \ll T_c$ is satisfied.

At nearly zero temperature, L probably has the integer value which is closest to (13). Increasing temperature presumably changes L by discontinuous steps as in the three-dimensional ANNNI model (Villain 1962, Elliott 1961, Fisher and Selke 1981, Bak 1982), until the temperature T_R given by (26) is reached. The alternative possibility of a devil's staircase (Aubrey *et al* (1985) and references therein) should also be considered. Above T_R the fluctuation $\langle (u_m^p)^2 \rangle$ which may be calculated from (25) is infinite and the wall network forms a floating solid (Nelson and Halperin 1979) as in the two-dimensional ANNNI model (Villain and Bak 1981, Bak 1982). Since a crystalline symmetry is imposed, no intermediate (hexatic or 'tetratic') phase (Nelson and Halperin 1979) is expected.

5. Conclusion

Two-dimensional Ising ferromagnets with spins perpendicular to the plane are destabilised by dipole interactions. Ferromagnetism is replaced by domain walls which probably form a square network on a square lattice. The period is unphysically large at low temperatures if the exchange energy is, say, more than 10 times the dipole energy. Near the transition, however, the period (at least as predicted by the mean-field approximation) should be measurable in all cases. The period varies continuously with temperature except at very low temperatures where, in a clean sample, a succession of commensurate structures is expected.

Experimental confirmation should be possible on magnetic adsorbed films, but the requirements are rather stringent: the system should be an Ising ferromagnet with spins perpendicular to the surface, and the exchange interaction should be weak. However,

the case of a finite anisotropy has probably similar properties (in particular, domain walls forming a square lattice) and a shorter period as shown by YG. However, the walls are thicker and commensurate structures even less stable.

Acknowledgment

We thank Professor Y Yafet for discussions and communication of his preprint before publication.

Appendix. Proof of (12)

We first consider the contribution W''_{dip} to (5) of those domains B_m which are not neighbours of A_0 (denoted $B_5, B_6, B_7, \ldots$). For any $i \in A_0$, there is the upper bound

$$\sum_{m\geqslant 5}\sum_{j\in B_m}\frac{1}{r_{ij}^3} < \sum_{r\notin\Gamma_i(L,M)}\frac{1}{r^3} \equiv Q(L,M)$$

where $\Gamma_i(L, M)$ is the rectangle of sides $2L \times 2M$ centred on i (figure 1). It results, after summation over i in (5), in

$$|W''_{\text{dip}}| < NgQ(L,M). \tag{A1}$$

Now, replacing sums by integrals gives

$$\begin{aligned} Q(L,M) &= 2\int_L^\infty \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}y\,(x^2+y^2)^{-3/2} + 2\int_m^\infty \mathrm{d}y \int_{-L}^{L} \mathrm{d}x\,(x^2+y^2)^{-3/2} \\ &= 4\int_L^\infty \frac{\mathrm{d}x}{x^2} + 4\int_M^\infty \mathrm{d}y \frac{L}{y^2\sqrt{y^2+L^2}} < \frac{4}{L} + 4\int_M^\infty \frac{\mathrm{d}y}{y^2} = \frac{4}{L}+\frac{4}{M}. \end{aligned} \tag{A2}$$

Insertion into (A1) yields

$$(1/A)|W''_{\text{dip}}| < 4g(L^{-1}+M^{-1}) = (g/LM)\mathrm{O}(\sqrt{L^2+M^2}).$$

Next, we consider the contribution K_1 of the neighbour domain B_1 (figure 1). In (10), K_1 is given as a sum of two integrals I_1 and I_2:

$$K_1 = 2(I_1 + I_2). \tag{A3}$$

Since the calculation of these integrals is rather tedious, we shall only sketch how the result (11) is obtained:

$$\begin{aligned} I_1 &= \int_{y_0}^{M} \mathrm{d}y\,(M-y)\left(\frac{1}{\sqrt{x_0^2+y^2}} - \frac{1}{\sqrt{L^2+y^2}}\right) \\ &= M[\ln(M+\sqrt{x_0^2+M^2}) - \ln(y_0+\sqrt{x_0^2+y_0^2})] \\ &\quad - M\{\ln[(M+\sqrt{L^2+M^2})] - \ln[(y_0+\sqrt{y_0^2+L^2})]\} \\ &\quad - (\sqrt{x_0^2+M^2} - \sqrt{x_0^2+y_0^2}) + (\sqrt{L^2+M^2} - \sqrt{L^2+y_0^2}). \end{aligned}$$

As one readily sees, the leading behaviour for $L, M \gg 1$ is independent of the lower bounds x_0 and y_0:

$$I_1 = M\ln[L/(1+\sqrt{1+L^2/M^2})] + \mathrm{O}(\sqrt{L^2+M^2}). \tag{A4}$$

We refrain from performing the integral I_2 explicitly. It is easily seen that I_2 is

$O(\sqrt{L^2+M^2})$ and can thus be neglected with respect to I_1 when $L, M \gg 1$. Equation (A4) inserted into (A3) proves equation (11). Use of (A1) then leads to

$$W/N = 2J(L^{-1}+M^{-1})f(\theta) - (4/L)g \ln L - (4/M)g \ln M + 4g\varphi(L,M) + (g/LM)O(\sqrt{L^2+M^2}) \tag{A5}$$

where

$$\varphi(L,M) = L^{-1}\ln(1+\sqrt{1+L^2/M^2}) + M^{-1}\ln(1+\sqrt{1+M^2/L^2}). \tag{A6}$$

Assume for instance that

$$L < M. \tag{A7}$$

Then (A6) implies that

$$\begin{aligned}|\varphi(L,M)| &< L^{-1}\ln(1+\sqrt{2}) + M^{-1}\ln[(M/L)(1+\sqrt{2})] \\ &< L^{-1}[\ln(1+\sqrt{2}) + (L/M)\ln(M/L) + (L/M)\ln(1+\sqrt{2})] \\ &< L^{-1}[1+2\ln(1+\sqrt{2})].\end{aligned}$$

This expression has the form $(g/LM)O(\sqrt{L^2+M^2})$. Thus (A5) reduces to (12).

Note added in proof. After this work was completed we became aware of the work of Garel and Doniach (1982) and Gabay and Garel (1985) who considered the dipolar Ising ferromagnet for the geometry of an infinite slab of finite thickness. The authors treated the problem addressed in the present paper and in addition investigated in great detail the effect of a field and of the thickness. However, we disagree on their results for zero field because we have shown that the domain walls form a square network rather than a striped structure as claimed by Garel and Doniach.

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